May, 2013

MATHEMATICAL BASIS OF LIFE ASSURANCE

[Time : 3 Hours] [Total Marks:100]

Answer ANY FIVE questions only. All questions carries 20 marks each.
(Candidates are allowed to refer Handbook on Formulae and Tables)

Marks 5 each

Q.1 a) Explain simple reversionary bonus system of distributing surplus and bring out the advantages of the system in comparison with other systems in vogue.

b) List the special reserves and adjustments provided to arrive at the final valuation liability.

c) Prove that the present value of Increasing annuity wherein the successive instalments form an Arithmetic progression

\[ A \cdot a_m + D \cdot V \left\{ \frac{a_m - n \cdot V^+}{i} \right\} \]

where 'A' is the first payment at the end of 1st year
'D' is the common difference between successive instalments
'n' is the term of annuity in years.

d) Provident Fund deductions are made monthly at a rate of ₹200/- per month and credited to P.F. account. Find the accumulated value at the end of 10 years, at a rate of interest of 10% p.a.

 Marks 5 each

Q.2 a) Explain New Business Strain.

b) Calculate the net annual premium limited to 15 years for a whole life assurance of ₹10,000/- on the life of (40). Assume that death benefit is payable immediately, on death.
Interest rate 6% p.a.; M_{40} = 17625.63;

\[ N_{40} = 1343014.73 \quad N_{55} = 403807.17 \]

What is meant by a policy value? Explain how this value arises?

Q.3 a) A whole life policy for ₹50,000/- effected at age 35. The policy has been in force for 15 years. The policy accrued a reversionary bonus of ₹13,500/-. Find the following using given data:

\[ N_{35} = 1906522.39 \quad N_{50} = 623195.21 \]

\[ D_{50} = 49929.83 \quad M_{35} = 18747.99 \quad M_{50} = 14654.66 \]

b) Establish the following relationship mathematically:

\[ a_x = 1 + a_x \]

\[ a_{x+n} = 1 + a_{x+n-1} \]

Q.4 a) \((tV_x + P_x) \times (1 + i) = q_{x+t} + P_{x+t} V_x\). Prove and Define the formula. What does it state?

b) i) Value of Assets over value of Liabilities is known as Surplus. What do you mean by surplus arising?

ii) What are sources of surplus in a Life Insurance Company?
Q.5   a) A mortality table is represented by the function :
\[ \lambda_x = 1000 \sqrt{100-x} \]
find (i) The probability of a life aged ‘0’ surviving to age 51
(ii) The probability of a life aged (36) dying before age (51)

b) Complete the following table

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>q_x</th>
<th>\lambda_x</th>
<th>d_x</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>0.33</td>
<td>30,000</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Define \( m;n q_x \)
And Prove; \( m;n q_x = m\xi - m+n\xi x \)

Q.6 a) Give expressions for the retrospective policy value and prospective policy value at the end of ‘t’ years under a pure endowment policy for a unit sum assured effected on the life of a person at age ‘x’ for term of ‘n’ years. Annual premiums under the policy are payable for a maximum of ‘n’ years. On the life assured surviving to ‘n’ years, the sum assured becomes payable; and on his death during the term of the policy, the total premium paid are returnable without interest.

b) The following particulars are given :

<table>
<thead>
<tr>
<th>Age ‘x’</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_x )</td>
<td>97380</td>
<td>97088</td>
<td>96794</td>
<td>96496</td>
<td>96194</td>
<td>95887</td>
</tr>
<tr>
<td>( d_x )</td>
<td>292</td>
<td>294</td>
<td>298</td>
<td>302</td>
<td>307</td>
<td>313</td>
</tr>
</tbody>
</table>

Calculate ignoring interest and expenses :
- The value of temporary assurance of ₹ 1,000/- for 2 years for a person aged 25.
- The value of endowment assurance benefit of ₹ 1,000/- for 4 years to a person aged 25.

Q.7 a) The subject knowledge of FL81, the exam you are appearing for, require knowledge of commutation functions. With the advent of sophisticated computers and softwares do you feel that commutation functions are still relevant as subject knowledge of FL81 subject. You need to explain in both the situations, whether your answer is ‘YES’ or ‘NO’.

b) i) An amount of 1 is paid at the end of year of death happening to the nominee of a person now aged(x). The amount of 1 would be paid if death happens in any of n years, means death during x to x+1 to x+n-1 to x+n. Nothing is paid on survival to x+n. Give actuarial notation for the present value of this benefit.
Rate of interest may be assumed i,

ii) Make use of probability of death (q_x) and probability of survival( p_x, p_{x+1}.....) and finally show b(i) formula in terms of commutation functions M(age), and D(age)
iii) How the formula shall look like if payment of 1 in b(i) is made on death and not at end of year of death. Establish relationship between this formula and as given in b(i)
c) A father wishes to provide an annuity of ₹10,000 per annum to his daughter, now aged exactly (15), the first payment being on her age (21). Ignoring expenses and profit margin what sum would an insurance company require as a premium, if it uses the following commutation functions and rate of interest (i)

\[ N_{21} = 4761494.14 \]
\[ D_{15} = 417265.09 \]
\[ i = 6\% \]

Q.8  a) ex represents expectation of life at age x, define it, and establish the relationship of curtate expectation of life with complete expectation of life. Use standard actuarial notations.
b) Define \( q_{[x]+1} \)
c) Establish relationship between rate of mortality (death rate) and Central death rate.
d) (i) What do you understand by ultimate mortality table, and aggregate mortality table, (ii) Name three methods of mortality investigation and briefly explain how each method is different from one another, (iii) In Assured lives mortality table, annuitant mortality table and census mortality table, how you expect mortality rate for life aged 36.
e) Which mortality table for Assured Lives has been prescribed by IRDA for use of Life Insurance Companies operating in India.

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