May, 2012

FELLOWSHIP EXAMINATION
MATHEMATICAL BASIS OF LIFE ASSURANCE

Time: 3 Hours

Answer FIVE Questions only.
All Questions carry 20 Marks Each.

Marks

Q.1. a) Prove that - :

\[ \nu_{x+\alpha} = \frac{P_{x+\alpha} - P_x}{P_{x+\alpha} + d} \]

b) What is meant by fair distribution of Surplus? Discuss how far this can be achieved?

c) Give expressions for the Prospective Policy Value and Retrospective Policy Value at the end of ‘t’ years under a Whole Life Policy by Single Premium for a unit Sum Assured, effected on the Life of a Person at Age ‘x’. Show that the two expressions are equal.

d) Calculate Net Annual Premium under a Special Temporary Assurance for ₹10,000 on (40) for 20 Years. On the Life Assured’s Survival to the end of 20 Years, total amount of Premiums paid will be payable.

[ Basis: LIC (1970 - 73) Table, 6% Interest:

\[ \bar{a}_{40:20} = 11.682, \]

\[ M_{40} = 17625.63 \]

\[ M_{60} = 10506.87 \]

\[ D_{40} = 93645.23 \]

\[ D_{60} = 24604.43 \]

Q.2. a) Define terms and explain by general reasoning the relationship :

\[(V_x + P_x)(1 + i) = q_{x+t} + P_{x+t}(\nu_{x+1} V_x)\]

b) What is Level Premium? Give advantages and consequences of charging Level Premium.

c) On basis of LIC (1970-73) Table at 6%, calculate Net Annual Premium ceasing after 15 Years or at previous Death for a Money Back Policy on (45) to secure the following Benefits:

i) ₹1,500/- on Survival to the End of 5 Years.

ii) ₹1,500/- on Survival to the End of 10 Years.

iii) ₹3,000/- on Survival to the end of 15 Years.

iv) ₹6,000/- on Death at any time within 15 Years.

(Given - \[ \bar{a}_{45:15} = 9.862, A_{45:15} = 0.72599, \]

\[ A_{45:10} = 0.51724, A_{45:15} = 0.35775, A_{45:15} = 0.08402 \]
d) Derive for Endowment Assurance:

\[ P_{x\mid n} = p_{x\mid n}^1 + p_{x\mid n}^1 \]

Q.3. a) A special policy provides for following benefits:

i) Initial Sum of ₹10,000/- with Guaranteed Annual Addition of ₹250/- for each Year's Premium paid after the first, if Death occurs within the Term of Assurance.

ii) ₹10,000/- payable on Survival to the end of the Term of Assurance.

iii) Free Paid-Up Assurance of ₹10,000/- Payable at Death after expiry of Term of Assurance.

Calculate net Annual Premium under the Policy on Life of (35) for 25 Years

[Basis: LIC (1970-73) Table, 6% Rate of Interest:

\[ M_{35} = 18747.99 \]
\[ M_{60} = 10506.87 \]
\[ D_{60} = 24604.43 \]
\[ D_{35} = 126664.23 \]
\[ R_{36} = 49758.69 \]
\[ \ddot{a}_{35\mid 25} = 13.086 \]

b) Calculate the Retrospective Policy Value at 6% Interest at the end of 3 Years for Temporary Assurance of ₹1000/- for a period of 6 Years on the Life of a Person aged 50. Use the following Mortality Table on the basis of which a Net Annual Premium of 9.26 is charged.

<table>
<thead>
<tr>
<th>Age</th>
<th>x</th>
<th>50</th>
<th>51</th>
<th>52</th>
<th>53</th>
</tr>
</thead>
<tbody>
<tr>
<td>1_x</td>
<td>920</td>
<td>913</td>
<td>905</td>
<td>896</td>
<td></td>
</tr>
</tbody>
</table>

Q.4 a) Explain why Life Insurers set up Reserves for the Endowment Assurance Policies.

b) Demonstrate the equality of Gross Premium Retrospective and Prospective Reserves for a Whole Life Policy, given the conditions necessary for equality.

c) A Temporary Annuity of ₹3,000/- p.a. payable Quarterly, in arrears, for a Term of 10 Years was purchased one year ago by Jim, on his 60th birthday, by payment of a Single Premium. Show algebraically that the current Retrospective and Prospective Net Provisions are equal assuming that the Premium and Provisioning Bases are the same. Ignore expenses.

Q.5. i) You are given

a) \[ A_x = 0.28 \]

b) \[ A_{x+20} = 0.40 \]
c) \[ A_{x \rightarrow 20} = 0.25 \]

d) \[ i = 0.05 \]

Calculate \( \bar{a}_{x \rightarrow 20} \)

ii) Establish, by general reasoning, the following relations:

a) \[ A_x = 1 - da_x \]

b) \[ A_{x \rightarrow 20} = 1 - da_{x \rightarrow 20} \]

Explain why this relation does not hold good in case of a Temporary Assurance.

iii) A Life Assurance Company issues Annual Premium Whole Life Assurance Policies with a Sum Assured of Rs. 1,00,000/- payable at the end of the year of death to lives aged exactly 35. Calculate the Premium using the Principle of Equivalence using the following basis:

Interest: 6% per annum

Expenses: 5% of premium

Given: \( \bar{a}_{[35]} = 15.993 \)

Q.6. On 1 January 2002, an insurer issued a block of 25-Year Annual Premium Endowment Policies that pays Rs. 1,20,000/- at Maturity, or Rs. 60,000/- at the end of the Year of Earlier Death to Lives aged exactly 65. The premium basis assumed 4% interest, select mortality and allow for an Initial Expenses of Rs. 200/- and Renewal Expenses of 1% of each subsequent Premium. Provisions are calculated on the same basis as on the Premiums.

i) Calculate the Premium.

ii) Calculate the Provision required per policy at 31st December 2006, independently using a Prospective and a Retrospective formula.

Given:

\[ A_{[65], 25} = 0.4658 \]

\[ A_{[70], 20} = 0.5218 \]

\[ A_{[65], 5} = 0.072 \]

\[ \bar{a}_{[65], 25} = 12.0455 \]

\[ \bar{a}_{[70], 20} = 9.989 \]

\[ \bar{a}_{[65], 5} = 4.5082 \]

\[ \frac{D_{90}}{D_{65}} = 0.0709 \]

\[ \frac{D_{90}}{D_{70}} = 0.094 \]
Q.7. i) Calculate Office Annual Premium for a Whole Life Assurance for ₹ 20,000/- to a person aged 40. Provide for First Year Expenses at 55% of Premiums and 17% of Sum Assured; and Renewal Expenses of 5% of Premium and 6% of Sum Assured. Basis: LIC (1970-73) Ultimate Table and 6% Interest.

ii) For the data of (i) above calculate Office Annual Premium for a With Profits Whole Life Assurance by providing a Bonus Loading of ₹ 25%o per annum.

Given:
\[ \bar{a}_{40} = 14.342 \]
\[ A_{40} = 0.18822 \]
\[ (IA)_{40} = 4.5360 \]

Q.8. i) Explain Simple Reversionary Bonus System of distributing Surplus and explain advantages of the system.

ii) What do you understand by Negative Values? How do they arise? What treatment is given to the Negative Values in a Valuation?

iii) Prove that: \[ V_x = \frac{P_x - P_{x+1}^1}{P_{x+1}^1} \]

END
★★★★★